The work thus covers the domain of general astronomy, with, however, one notable exception, namely, the omission of all reference to new stars! The reader is thus left entirely ignorant not only of the facts that such bodies as Nova Aurigæ, Nova Persei, Nova Geminorum, &c., ever existed, but of the various hypotheses put forward to explain the sequence of the interesting and important phenomena which are so characteristic of them.

This omission is, however, not the only blot which mars this book, for unfortunately errors of another kind are by no means uncommon.

Those who have taught astronomy know how important it is to give the student a correct idea of the difference between "rotation" and "revolution," so that the beginner may clearly grasp the facts that the former is responsible for our day and the latter for our year.

For a popular work, such as this, the definition of rotation could scarcely be more clearly explained than is done under the heading "rotation" in Webster's Dictionary (1902):—"The act of rotating or turning, as a wheel or a solid body on its axis, as distinguished from the progressive motion of a body round another body or a distant point. Thus, the daily turning of the earth on its axis is a rotation; its annual motion round the sun is a revolution."

With these definitions before us the following extracts from the book under review may be of interest. On p. 11, for instance, the reader is told that "the earth is not at rest, but *revolves* unceasingly around an axis . . .," and on the same page that "this real *revolution* of the earth, with the apparent revolution of the stars which it causes, is called the *diurnal motion*. . . ." Again, on p. 19 we read, "as the earth *revolves* on its axis. . . ."

If the author had expressly stated that his definition of "rotation" referred to *points* on the earth's *surface* and not to the earth as a whole, then the above statements might be valid, but as he makes no mention of this, the beginner will undoubtedly become perplexed as regards these motions.

A little further on (p. 35) a description is given of how the obliquity of the ecliptic produces the changes of seasons. Unfortunately (line 9) the word "orbit" is printed instead of "axis," an error which by no means renders the explanation very clear.

Another difficulty which the beginner will have to overcome occurs on p. 57, where the illustration showing the axes on which a telescope turns is placed on its side. Apropos of the incorrectness of diagrams, an error occurs in the drawing of the path of the rays (p. 68) illustrating the principle of the Newtonian reflecting telescope. Here the "flat" or "secondary mirror" is placed outside the focus of the large reflector, so that the rays which after reflection from the latter fall on it are divergent and not convergent.

In this chapter it is stated that "the largest mirrors so far successfully made and used have been about four feet in diameter." The author does not seem to be aware that the late Dr. Common constructed, mounted, and used a mirror measuring five feet in diameter.

It might also be suggested here that the diagram of the solar spectrum (p. 75) should be placed horizontally and not vertically, as this latter position would tend rather to confuse than to enlighten beginners when they are confronted later with terrestrial or celestial spectra.

On p. 114 a rather perplexing statement is made:—

"if we imagine ourselves standing *exactly* on a pole of the earth, with a flagstaff fastened in the ground, we should be carried round the flagstaff by the earth's rotation. . . ."

To the writer of this notice it seems that the flagstaff would travel round the observer if the observer be standing *exactly* on a pole of the earth as is stated; of course, it is meant that the flagstaff should be placed on a pole and the observer near it, but the reader has good cause to be puzzled.

A point which calls for special attention when giving our readers an idea of the contents of this book is the extreme poorness of the illustrations. One would have thought that advantage would be taken of the wealth and excellence of astronomical photographs that are now available, and the facility and accuracy with which they can be reproduced; but this is not the case.

Sun-spots are represented by a single drawing made many years ago; comets are illustrated by four drawings made by G. P. Bond, instead of by some of the beautiful photographs secured at recent appearances. Further, Bond's drawing of Donati's comet is so badly reproduced that probably the original artist would not be able to recognise it; the frontispiece, an impression of the solar corona of 1900, is decidedly feeble. The reader is not shown either a stellar spectrum or a reproduction of Hale's fine spectroheliograph photographs, or even a spectroscope or objective prism telescope.

From the above remarks it will be gathered that the book before us is not the best that could be placed in the hands of a beginner, and it seems a pity that more trouble was not taken in its production.

HISTORY OF ELEMENTARY MATHEMATICS.

Geschichte der Elementar-mathematik in systematischer Darstellung. By Dr. Johannes Tropfke. Erster Band. Pp. viii+332. (Leipzig: Veit and Co., 1902.) Price 8 marks.

THE great work of Moritz Cantor has made him, as it were, the Gibbon of mathematical history. But the extent of his subject has prevented him, as a rule, from entering into detail, and there are many things of great interest about which it is not easy to get information without laborious research. The history of mathematics is being studied, and its value is recognised, not only by those who make it their special domain, but by an increasing number of practical teachers, so that there is both a demand for books dealing with various parts of the subject in different degrees of detail and a school of historians ready to supply them.

Dr. Tropfke's work is not exactly a popular treatise. He has limited himself to the range of elementary

mathematics, and in this volume deals only with arithmetic and algebra; but his treatment is thorough, and his aim has been to give exact references to the original authorities for the statements in the text. The amount of labour that this has involved must have been very great; when the work is complete, with the indexes promised by the author, it will be a valuable repertory for those who wish to learn the facts at first hand. The number of bibliographical footnotes exceeds 1200, and since many of these give more than one reference, it will be seen how great a service the author has rendered to those who are inclined for research.

But the book is far from being a mere dry collection of facts and references. The style is concise, and there is no catchpenny rhetoric, but there is plenty to interest any intelligent reader. The arrangement allows us to trace in detail the development of methods and of notation; we are shown, with explanations, the actual symbols used and the processes employed by our predecessors; most important of all, there is an appendix with a selection of original examples ranging from Alchwarizmi to Leibniz and Newton. Few things are more instructive than an inspection of some of the older methods in arithmetic. Until the end of the fifteenth century, long after the decimal notation and the use of the "Arabic" numerals had become familiar, and when arithmetical calculations were usually worked on paper, the rule for performing long division was of a most complicated character, with rows of figures above the dividend as well as below, and tedious cancellings and substitutions which must have made the operation both laborious and liable to error. It is almost certain that the process is of Indian origin, and it is probable that the figures which, in written examples, we find cancelled by a stroke drawn through them represent digits which were actually obliterated at an earlier period, when the calculation was performed with a stick on a layer of sand.

A striking feature of early European books on arithmetic is the bewildering number of their so-called "rules." One reason for this is simple enough. Many of these books were intended to help business men—bankers, merchants, and so on—in such calculations as their calling obliged them to do. Their interest in arithmetic was purely practical, and all they wanted was a bundle of recipes for getting correct answers to questions of certain special types. Even in our own day we occasionally see such terms as "agricultural book-keeping" or "chemical arithmetic," which show that a demand for this sort of thing is not yet extinct. But even in treatises of a more theoretical kind *duplatio* and *mediatio*, in other words doubling and halving, were reckoned as separate rules. This is a historical survival, a sort of fossil relic of prehistoric times. It appears that the ancient Egyptians performed multiplication by a process practically equivalent to converting the multiplier into the binary scale; thus

$$x \times 13 = x \times 8 + x \times 4 + x,$$

where $x \times 8$ and $x \times 4$ were obtained by successive doubling. When an improved method of multiplication had been discovered, the older process became obsolete; but *duplatio* held its ground as a special rule, in recognition, so to speak, of its former importance.

A considerable portion of this volume is naturally devoted to the theory of surds, and this cannot be separated from the Greek theory of geometrical irrationals. After all that has been written on the subject, *lacunae* remain which will probably never be filled up, unless new documents are discovered. Some undoubted facts are very puzzling when taken in combination. For instance, Euclid says in so many words that incommensurable quantities are not related to each other as numbers, and it really does seem that to a Greek geometer of Euclid's time the relation, as to length, of the diagonal of a square to one side was something different *in kind* from the relation of two commensurable distances. At the same time the Greeks must have been practically acquainted with what we should call rational approximations to $\sqrt{2}$, and it is well known that the irrationalities considered in the tenth book of Euclid's "Elements," when put into an algebraic form, correspond exactly to all the members of a particular group of surds, without omission or redundancy. Did the geometers, who professed to despise "logistic" in public, privately make use of it to help them in their researches?

Other subjects considered under the head of algebra are the development of the idea of number in general, the operations of algebra and their symbols, proportion, and equations. Under the last heading Diophantine analysis is included, and it may be noted as a fact not generally known that Diophantine equations of the form

$$px^2 - qy^2 = r$$

were actually discussed in India at least as early as the time of Brahmagupta—that is to say, more than a thousand years before Fermat proposed the Pellian equation to the English mathematicians. G. B. M.

OUR BOOK SHELF.

La Lutte pour l'Existence et l'Évolution des Sociétés. By J. L. de Lanessan. Pp. 277. (Paris: Félix Alcan, 1903.) Price 6 francs.

THE title of this book is most misleading. The reader naturally expects to find an account of the struggle for existence among primitive men and of the evolution which has resulted from the struggle. The first chapter has quotations from Buffon and Darwin which leave no doubt in one's mind that this is the line which is to be followed. After this comes a description of primitive society or rather the social system which the author assumes to be primitive. The struggle for existence drops out, and is not mentioned. Society begins, he tells us, with a severely patriarchal *régime*. He seems not to have heard of an earlier polyandrous period. Out of the family bond arose the sense of duty. Speaking of the tribe, he lays it down that the chieftain was regarded as the owner of all the land which the tribe possessed.

After this glance at primitive society, we plunge into French history. Many great questions are dealt with, and most of them with remarkable shrewdness. Our author discusses the origin of feudalism. He next decides that Christianity had nothing to do with the abolition of slavery. He traces the growth of the idea of liberty among the peasantry; it showed itself